18.312: Algebraic Combinatorics Lionel Levine

Problem Set 6

Due at the beginning of class on Thursday April 14, 2011

P26

(a) Let C_n be the cycle graph with vertex set $[n]$ and edge set $\{(1,2),(2,3),\ldots,(n-1,n),(n,1)\}$. Find the chromatic polynomial of C_n.

(b) Let $H_{ij} = \{(x_, \ldots, x_n) \in \mathbb{R}^n \mid x_i = x_j\}$. Find the number of connected components of $\mathbb{R}^n - (H_{12} \cup H_{23} \cup \ldots \cup H_{n1})$.

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P27 Let X be a set of cardinality n, and let $0 \leq k \leq n/2$. Prove that there is a bijection f from subsets of cardinality k to subsets of cardinality $n-k$

$$f : \left(\begin{array}{c} X \\ k \end{array} \right) \to \left(\begin{array}{c} X \\ n-k \end{array} \right)$$

such that $S \subset f(S)$ for all sets $S \in \left(\begin{array}{c} X \\ k \end{array} \right)$.

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P28 Let $G = (V, E)$ be a graph (not necessarily bipartite) such that $|\Gamma(A)| \geq |A|$ for all $A \subset V$. Prove that there exists a permutation σ of V such that $(v, \sigma v) \in E$ for all $v \in V$.

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P29 Let $k \in \mathbb{N}$, and let $G = (V, E)$ be a k-regular bipartite graph (every vertex lies on k edges). Prove that the edge set E can be partitioned into k disjoint perfect matchings.